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Optimal Design of Nonlinear Minimum Phase Error IIR Digital Filter with Prescribed Magnitude and Phase Constraints

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Abstract— Infinite Impulse response (IIR) digital filters are used widely in many applications. In this paper, a new approach to the design of general IIR filters by minimizing the maximum phase error subjected to prescribed or simultaneously minimized maximum magnitude error. Here the desired frequency response is acquired by prescribed magnitude and phase responses. The sequential constrained Least Square method and Levy-Sanathanan-Koerner strategy are used to convert the non-convex constraints into convex constraints. The sequential constrained least-squares (SCLS) method has a higher possibility of obtaining better solutions than a direct minimization method when applied to the non-convex minimax design of IIR filters**.** The MMPE design of frequency selective filters subject to prescribed or simultaneously minimized maximum magnitude error is also considered.

Keywords: Optimization, L-SK strategy, MMPE method, ECG noise removal.

I.INTRODUCTION

Digital filters such as Infinite impulse response (IIR) digital filters with prescribed magnitude and phase responses have been used in many applications, such as digital communications, phase equalization, implementation of digital and multirate filters, etc. [1], due to their low implementation complexity and good numerical property. The major difficulties for designing an IIR digital filter are its nonlinearity and stability problems. Large number of procedures are available for designing IIR digital filters which approximate the given magnitude and phase responses simultaneously. To approximate a given frequency response with prescribed magnitude and phase characteristics by a general IIR filter,the minimization of maximum frequency response error (MMFRE) [2], without imposing constraints on any other types of errors. Since the frequency response error constraint is circular in the complex plane of frequency response, maximum phase errors obtained by these methods are almost as large as maximum magnitude errors. The elliptic error constraint with a major axis along the desired frequency response has also been used in the design of two-dimensional FIR filter with prescribed phase error to reduce the magnitude error [1], with reducing the group delay also.

A major challenge in the optimal design of infinite impulse response (IIR) filters is the nonconvexity of the resulting optimization problem. For constrained least-squares (CLS) and least p-power error designs, several algorithms based on modified versions of the Steiglitz–McBride (SM) strategy [3] were presented to convert the nonconvex problems into a series of standard convex problems such as quadratic programming (QP) [4]-[6], , and second-order conic programming (SOCP) [7] problems. In another category of design methods, the minimax problem was converted into a sequence of minimization problems of other type. The design procedure in [4], used an iterative reweighting technique [7] to convert the minimax design into a sequence of weighted LS problems, which were then solved by the multistage method [3]in which both magnitude and phase are optimized using a weighted and sampled least-squares criterion. A new convex stability domain defined by positive realness for ensuring the stability of the filter and adapt the Steiglitz–McBride (SM), Gauss–Newton (GN), and classical descent methods to the new stability domain.. The design algorithm in [4] used a bisection technique to iteratively locate the minimum FR-error upper bound and to get in each iteration a nonconvex feasibility problem, which was then solved by relaxing into a semi-definite programming (SDP) problem.

An iterative linear programming approach is presented to design stable IIR digital filters with prescribed magnitude and phase responses [2]. At each iteration, the complex error of the frequency response is transformed into a linear form by treating the denominator polynomial obtained from the

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preceding iteration as a part of the weighting function, and the poles restricted inside the unit circle by using a set of linear constraints. After solving the standard linear programming problem at each iteration, the design algorithm converges to the minimax solution which proved to be a better design results than the conventional linear programming method.

II.PROPOSED METHOD

In this paper, two elliptic constraints are imposed on the frequency response of an IIR filter, one to minimize the maximum phase error, and the other to constrain the maximum magnitude error. Several methods have been proposed to convert non-convex circular frequency response error constraints into convex ones. Levy–Sanathanan – Koerner (L-SK) strategy is used in this paper to convert the non-convex equations to convex ones. This method replace the denominator of the frequency response error constraint with an estimate obtained in the previous iteration .

Using L-SK strategy, we also replace the denominators of the two elliptic constraints with their previous estimates. The resultant constraint for magnitude error is convex, but the constraint for phase error is still non-convex. To this end, we combine the L-SK strategy with a sequential constrained least-squares (SCLS) method, resulting in a convex optimization problem.

III. METHODOLOGY

The main purpose of this project is to design a digital IIR filter with minimum phase error, find the coefficient values of the designed filter and approximate it with the desired frequency response. As an input we have our desired frequency response as a array of values. So we need to approximate a given frequency with prescribed magnitude and phase response characteristics by designing a digital IIR filter with minimum phase and magnitude error. First of all we need to approximate a desired frequency response D(w) by the actual frequency response $G(e^{jw})$ of the IIR filter. The G(w) is assumed and is taken as any type of the classical IIR filters. Here we represents the digital angular frequency and j is the imaginary unit.

3.1. Formulation of Constrained Least Square Design for the Phase and Magnitude Error.
Now we need to formulate the complex error and phase error constrained least squares design.

The frequency response error is given by:

$$E(w) = G(e^{jw}) - D(w) \qquad (1)$$

The corresponding magnitude and phase error is given by:

$$E_m(w) = |G(e^{jw})| - D(w) \qquad (2)$$

$$E_p(w) = \alpha(w) - \Phi(w) \qquad (3)$$

$\alpha(w)$ & $\Phi(w)$ are the phases of the $G(e^{jw})$ and D(w).

The frequency response error can be approximated by setting an upper bound ρ:

$$|E_m(w)| = |G(e^{jw})| - |D(w)| \qquad (4)$$

Where Ω_p and Ω_s represent the passband and stopband of the filter.
The boundary of the above constraint at an arbitrary frequency is circular in the complex plane of $G(e^{jw})$, as shown by the dotted circle in Fig.3.1.

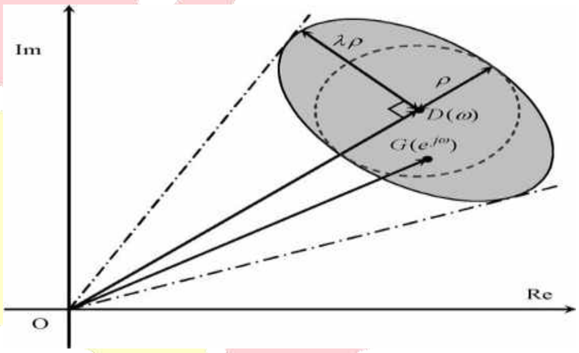

Fig.3.1.Frequency response error constraints

From the figure we define a transformed frequency error response error as:

$$\bar{E}(w) = e^{-j\Phi(w)} E(w) \qquad (5)$$

From the fig:1, we define a transformed frequency response error as:

$$|Re[\bar{E}(w)] + j\lambda\, Im[\bar{E}(w)]| \le \rho \qquad (6)$$

Here ρ is the magnitude error

- Where λ > 0 is the ratio of the major axis to minor axis of the ellipse.
- If λ >1,the constraint (6) imposes a tighter limit on the magnitude error than the phase error and vice versa.
- If λ =1,the ellipse degenerates to a circle and the elliptical constraint reduces to the circular constraint.

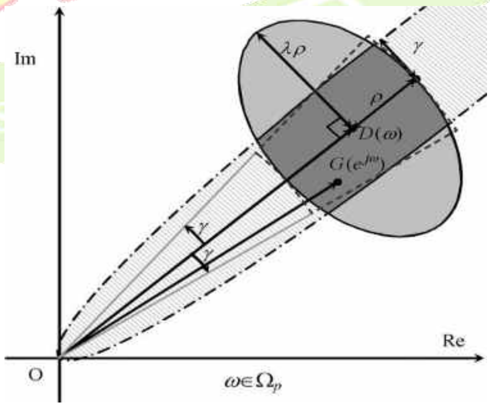

Fig.3.2. Frequency response error constraints.

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In order to control the magnitude and phase error more flexibly, we consider another frequency response with ellipse centered at $[|D(w)| + \rho]\, e^{-j\Phi(w)}$,major axis of length $|D(w)| + \rho$ along D(w) and a minor axis of length γ as shown in fig 2.

$$\check{E}(w) = e^{-j\Phi(w)} G(e^{jw}) - [|D(w)| + \rho] \quad (7)$$

The error constraint defined by the dotted ellipse is:

$$\left|\frac{\gamma Re[\check{E}(w)]}{|D(w)|+\rho} + jIm[\check{E}(w)]\right| \leq \gamma \quad (8)$$

If the ρ and γ are very small then its intersection of ellipses is very close to exact sector domain of the magnitude error and phase error constraints $|E_m(w)| \leq \rho$ and $|E_p(w)| \leq \gamma$.From the constraints (6) and (8) we can control the maximum magnitude error and phase error .

The magnitude error and phase error can be controlled by the equations;

$$\check{E}_m(w) = |Re[E(w)] + j\lambda\, Im[E(w)]| \quad (9)$$

$$\check{E}_p(w) = \gamma \frac{Re[\check{E}(w)]}{|D(w)|+\rho} + jIm[\check{E}(w)] \quad (10)$$

Now we consider an digital IIR filter;

$$G(z) = \frac{B(z,b)}{A(z,a)} \quad (11)$$

B(z,b) and A(z,a) are the numerator and denominator polynomials of the filter's transfer function described by:

$B(z,b) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots\ldots\ldots + b_M z^{-M}$
$A(z,a) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots\ldots\ldots + a_N z^{-N}$
Where $a = [a_1, a_2, \ldots\ldots, a_N]^T$ and $b = [b_0, b_1, b_2, \ldots\ldots, b_M]^T$
Now, introduce two complex function vectors:
$n(w) = e^{-j\Phi(w)} [1, e^{-jw}, \ldots\ldots, e^{-jMw}]^T$
$d(w) = -|D(w)|[e^{-jw}, e^{-j2w}, \ldots\ldots, e^{-jNw}]^T$

Thus, the frequency response :

$$G(e^{jw}, b, a) = \frac{B(e^{jw}, b)}{A(e^{jw}, a)} = e^{j\Phi(w)} n^T(w) b / A(e^{jw}, a) \quad (12)$$

By considering the new filter coefficients, we redesign the equations for transformed response error $\bar{E}(w)$, magnitude error $\check{E}_m(w)$, phase error $\check{E}_p(w)$ as:

$$\bar{E}(w, b, a) = \frac{n^T(w) b}{A(e^{jw}, a)} - |D(w)| = \frac{\bar{E}_n(w,b,a)}{A(e^{jw}, a)} \quad (13)$$

$$\check{E}_m(w, b, a) = Re\left[\frac{\bar{E}_n(w,b,a)}{A(e^{jw}, a)}\right] + jIm\left[\frac{\bar{E}_n(w,b,a)}{\lambda A(e^{jw}, a)}\right] \quad (14)$$

$$\check{E}_p(w,\gamma,b,a) = Re\left\{\frac{\gamma \check{E}_n(w,\rho,b,a)}{[|D(w)|+\rho] A(e^{jw}, a)}\right\} + jIm\left\{\frac{\check{E}_n(w,\rho,b,a)}{A(e^{jw}, a)}\right\} \quad (15)$$

Where,

$\bar{E}_n(w,b,a) = n^T(w) b + d^T(w) a - |D(w)|$
$\check{E}_n(w, \rho, b, a) = n^T(w) b + \tilde{d}^T(w, \rho) a - [|D(w)| + \rho]$
$\tilde{d}(w, \rho) = -[|D(w)| + \rho]\,[e^{-jw}, e^{-j2w}, \ldots\ldots, e^{-jNw}]^T$

The above equations are in non -convex state, so in order to convert it into convex state ,we apply the LSK strategy to the denominator by replacing the denominator $A(e^{jw}, a)$ with an estimate in the iterative procedure.

3.2. LSK Strategy

Here we assume the procedure is currently in the k^{th} iteration and the coefficient vector a obtained in the (k-1)th iteration is a(k-1).By the LSK strategy, we replace the denominator by its estimate $A_{k-1} = A(e^{jw}, a(k-1))$.

The transformed frequency error;

$$\bar{E}(w, b, a, k) = \frac{n^T(w) b + d^T(w) a - |D(w)|}{A_{k-1}(w)} \quad (16)$$

Modified magnitude error is:

$$\check{E}_m(w, b, a, k) = Re\left[\frac{\bar{E}_n(w,b,a)}{A_{k-1}(w)}\right] + jIm\left[\frac{\bar{E}_n(w,b,a)}{A_{k-1}(w)}\right] \quad (17)$$

Modified phase error is:

$$\check{E}_p(w,\gamma,b,a,k) = Re\left\{\frac{\gamma \check{E}_n(w,\rho,b,a)}{[|D(w)|+\rho] A_{k-1}(w)}\right\} + jIm\left\{\frac{\check{E}_n(w,\rho,b,a)}{A_{k-1}(w)}\right\} \quad (18)$$

The iteration is continued untill the constraints:

$$|\bar{E}(w, b, a, k)| \leq \rho$$

$$|\check{E}_m(w, b, a, k)| \leq \rho$$

$$|\check{E}_p(w, \gamma, b, a, k)| \leq \rho$$

Thus, if this criterion is satisfied, the equations will be in convex state.

3.3. MMPE method

3.3.1 MMPE with Prescribed Magnitude Error (MMPE-PME)

In MMPE method, we preset a magnitude error bound ρ which is subjected to the constraints:

$\min \delta$
s.t: $|\check{E}_p(w, \gamma(w), \rho, b, a)| \leq \gamma(w) = W_p^{-1}(w)\, \delta$
$|\check{E}_m(w, b, a)| \leq \rho$
$|\bar{E}(w, b, a)| \leq \rho$

$W_p(w) > 0$ is a phase error weight function
$\delta, b, a \in R(r)$,where R(r) is the filter's stability domain.

By applying LSK strategy to the subjected constraints and replacing the denominator we convert it into convex constraints.

$\min \delta$
s.t: $|\check{E}_p(w, \gamma(w), \rho, b, a, k)| \leq \gamma(w) = W_p^{-1}(w)\, \delta$
$|\check{E}_m(w, b, a, k)| \leq \rho$
$|\bar{E}(w, b, a, k)| \leq \rho$

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Now in the above constraints, 1st one is not in convex form because $\gamma(w)$ varies with decision variable δ. Inorder to convert this a method called Sequential Constrained Least Square(SCLS) is defined.

3.3.1.1. SCLS method

The SCLS method starts from a sufficiently large upper bound ρ_0, then reduces ρ_n by a factor of $\theta<1$ in each successive iterative step until the CLS problem is feasible in the n^{th} iteration with but infeasible in the (n+1)th iteration with $\rho_{n+1}=\rho_{min}$, and finally uses a bisection technique to search in the interval[ρ_{min}, ρ_{max}] for a minimum upper bound under which problem is feasible.

Now in this, we denote by[$\delta_n|n=0,1,\ldots..n$] an upper-bound sequence produced by the SCLS method, and let $\gamma_n(w) = \delta_n / W_p(w)$ and $\rho_n = \max(\delta_n , \rho)$.If we use generalized positive realness condition for describing the stability constraint a $\in R(r)$, then the core sub problem can be solved by MMPE-PME method which is given by:

$$\min 0.5(a^Ta + b^Tb)$$

s.t.: $| \check{E}_p (w, \gamma_n(w), \rho_n, b, a,k)| \leq \gamma_n(w)$

$| \check{E}_m (w, b, a,k)| \leq \rho_n$

$| \bar{E} (w, b, a,k)| \leq \rho_n$

$\mathrm{Re}\{e^{-j\psi_{k-1}(w)} A(re^{jw},a)\}>\varepsilon$

$\varepsilon >0$ is sufficiently a small number.

Since all the constraints are in convex form we can solve this by Goldfarb-Idnani algorithm for Complex-error and Phase-error Constrained Least-Squares (CPCLS-GI).

3.3.1.2. Goldfarb-Idnani algorithm.

For the finite positive-definite QP subject to , , Goldfarb and Idnani presented a very efficient and stable primal-dual algorithm . The algorithm starts with the unconstrained minimizer of the problem, and successively adds the most violated constraints to an active set until a solution is found. Since the number of constraints,M , is finite, the most violated constraint at an iterative point can be simply identified by the index υ satisfying:

$A_\upsilon^T - b_\upsilon = \max(A_1^T - b_1, A_2^T - b_2, \ldots\ldots.., A_M^T - b_M)$, i.e., we choose as the most violated constraint. In each step, the minimizer of the objective function subject to the new active set of constraints is computed. If an iterate satisfies all constraints, the optimal solution is found, and the algorithm terminates. If necessary, a constraint can be dropped from the active set if no longer considered as an active one.

3.3.2 MMPE-PME Design Algorithm

The solution starts by setting $\delta_e^* \geq 0$ be the expected value of minimax phase error δ^*,and LS solution be (b_{LS},a_{LS}) based on LSK method with Zero initial condition:

$$(b_{LS};a_{LS}) = \arg\min \sum |E(w)|^2$$

The procedure for MMPE-PME design is described as:

Step 1) Let $b(0)= b_{LS}, a(0)= a_{LS}$ and

$\delta_0 = \mu \max = | G(e^{jw}, b(0),a(0)) - D(w)|$

where μ is a real number chosen to ensure δ_0 to be sufficiently large. Let

$\delta_{max}= \delta_0$, $\delta_{min}= \delta_e^*$, $k=0, n=0, k_0=0$.

Step 2) Let $\gamma_0(w)= \delta_0/W_p(w)$ and $\rho_0=\max(\delta_0, \rho)$

Step 3) Let k=k-1. Solve problem (11) for b(k) and a(k) using CPCLS-GI. If (11) is infeasible,

let $\delta_{min}= \delta_n$,b(k)=b(k-1),a(k)=a(k-1) and go to Step 5.

Step 4) If $\| a(k)-a(k-1)\| > \Delta\|a(k-1)\|$ and $(k-k_n) < K$, where $0< \Delta<1$ is a small real number and $k>0$ is a sufficiently large integer, go back to Step 3; otherwise, let $\delta_{max}= \delta_n$.

Step 5) If $(\delta_{max}-\delta_{min}) \leq \upsilon\, \delta_{min}$, where $0< \upsilon <1$ is a small number defining the design tolerance, terminate.

Step 6) Let $n=n+1, k_n=k$. If $\delta_{min} = \delta_e^*$, let $\delta_n = \theta\, \delta_{n-1}$, where $0<\theta<1$ is a given shrinking factor.

Otherwise, let $\delta_n = 0.5(\delta_{max}+\delta_{min})$.

Step 7) Let $\gamma_n(w)= \delta_n/W_p(w)$, $\rho_n=\max(\delta_n, \rho)$. Go to Step 3.

The above procedure has an outer loop (SCLS) consisting of Steps 2–7 and an inner loop (L-SK) consisting of Steps 3–4.

It can be shown that the SCLS loop terminates after:

$$n_{max}=\mathrm{int}\left[\frac{\ln\left(\frac{\delta_e}{\delta_0}\right)}{\ln\theta}\right] + \max\left\{0, \mathrm{int}\left[\frac{\ln\left(\frac{1-\theta}{\varepsilon\theta}\right)}{\ln 2}\right]\right\}$$

outer iterations. The L-SK loop may not converge for some ρ_n and γ_n .To this end, we regard the ρ_n and γ_n as feasible and get out from the L-SK loop if K successive problems (11) are feasible. Therefore, the above procedure terminates within $K.n_{max}$ inner iterations.

IV. RESULT

The result obtained in designing the IIR filter with prescribed magnitude and phase responses is given below. Here the order taken is M=N=6.

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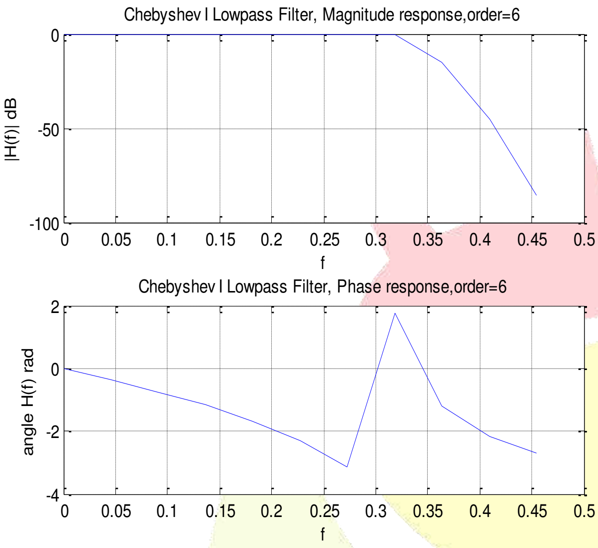

Fig: 4.1 IIR filter response

V. APPLICATION IN ECG SIGNAL RESTORATION

It is well known that biomedical signals carry important information about the behavior of the living systems under study. With the analysis of the Electrocardiogram (ECG) signal it may be possible to predict heart problems or monitor patient recovery after a heart intervention. A proper processing of these signals enhances their physiological and clinical information. The quality of biomedical signal is degraded mainly by many sources of noise such as power line interference (PLI), baseline drift, muscle contraction etc. The designed Chebyshev Type I digital filter of this paper can be used to overcome degradation by improving ECG signal quality for quality clinical diagnosis. Removing noise from the biomedical signal is still challenging and a rapidly expanding field with a wide range of applications in ECG noise reduction.

Fig.6.1 shows the basic ECG waveform with schematic representation of a single cycle of ECG corresponding to one heart beat.

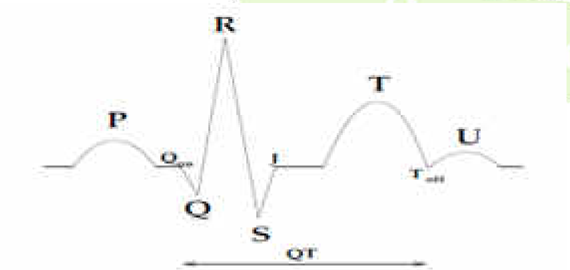

Fig 5.1: Schematic representation of a single cycle of ECG corresponding to one heart beat.

ECG Data Signal 222txt (ML II) take from Physionet Bank ATM as a input signal in analysis of removing noise by using IIR Filter Design techniques.

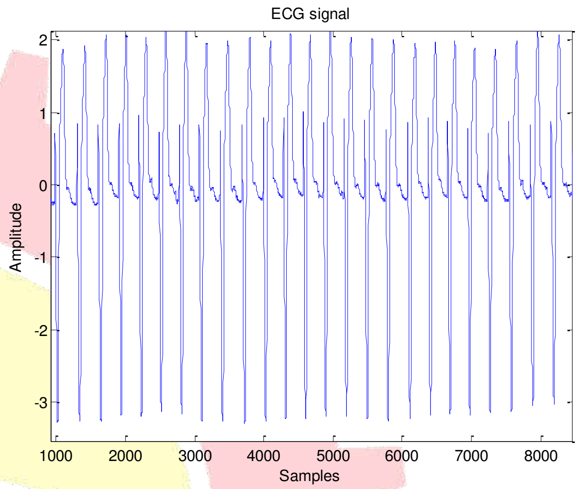

Fig: 5.2 ECG Signal

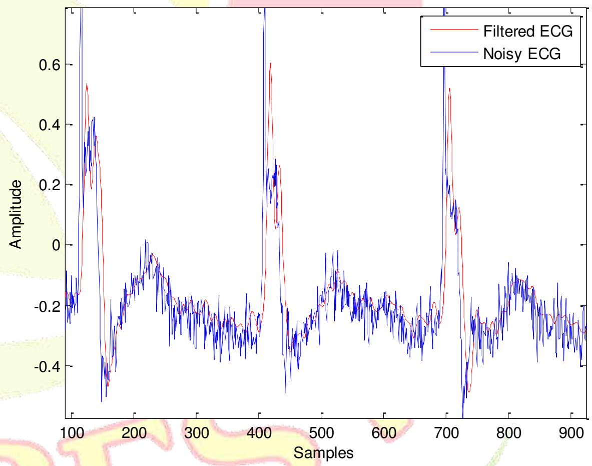

Fig: 5.3 Noisy and filtered ECG signals

From the result, it is seen that the filters reduces the low and high frequency components. The power line noise is also reduced.

VI. CONCLUSION

This paper has presented two minimax phase error design methods that aim at minimizing the maximum phase error of an IIR filter, subject to an upper bound constraint on the magnitude error or simultaneous minimization of the maximum magnitude error. In both methods, the magnitude and phase errors are controlled by two elliptic constraints on the frequency response, which are both nonconvex in the coefficient vector space. The nonconvex constraints are converted into convex ones by using the SCLS method and L-SK strategy. Design examples have shown that the minimax phase error design methods have obtained smaller or sometimes much smaller phase errors and group delay errors

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than the PCLS, SCLS, and AP-based design methods, sometimes at the sacrifice of a slight increase of magnitude error. Also, the minimax phase error design methods may obtain smaller transition-based magnitude overshoot than the SCLS method. Finally an application in ECG signal restoration is showed.

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