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A green production inventory model for multi-replenishment cycle and internally deteriorating items allowing shortages

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Abstract: This paper presents a multi imperfect-replenishment cycle inventory model wherein a single manufacturer produces items in *m* replenishment cycle and reworks the imperfect items, produced in *m* replenishment cycle, in one ideal rework setup. Rework is one of the significant issues in reverse logistic and green supply chain to reduce production cost and ecological issues. The rework process considered here for imperfect items starts after *m* replenishment cycle. The impact of deterioration cannot be disregarded as it affects the inventory system. In general, all the deteriorating items are identified from the inventory during screening process. But it is failed for some internally deteriorating food commodities. In this model, we consider imperfect inspection process of deteriorating items which leads to deliver deteriorated items to customers which will create negative impact on corporate image. Shortages are permitted and fully backordered. The objective of the paper is to determine the optimal replenishment run time and to minimize the total inventory cost of the manufacturer. Finally, a numerical example has been considered to illustrate the proposed model and then some sensitivity analyses have been carried out to get the impact of some parameters on the objective function of the proposed model.

Keywords: *Multi-replenishment cycle, Replenishment run time, Penalty cost, Complete backlogging, Rework.*

I. INTRODUCTION

Nowadays, the facts like deterioration, shortages and rework of defective items in the market, are rising very interest. Another major issue, in the global market, is how to reduce environmental pollution. Rework is one of the main issues in reverse logistic and green supply chain since it can reduce production cost and environmental problem. Many researchers focused on developing EPQ model for deteriorating items but few of them developed model for rework of defective items. In this paper, we develop an EPQ model for multi-replenishment cycles allowing shortages, considering penalty cost for selling deteriorating items to customers and rework of defective items to reduce environment pollution. In reality, production processes are often imperfect so that the production of imperfect items is unavoidable. For economic and environmental reasons, imperfect quality items are reworked to become serviceable items again. Due to unsuitable inventory condition or other reasons, the remaining good quality items stored in inventory are deteriorating. In order to provide good service to customers, inspection is carried out to screen out imperfect items. However, such inspection may not be perfect so that a portion of defective items produced are not successfully screened out internally during the screening process and passed on to customers, thereby causing defect sales returns and reverse logistics from customers back to the manufacturer. One common source of inspection error is from human factors (Drury [1] and Drury et al. [2]). The remaining perfect items will then be sold to customers. All the imperfect items are reworked as good quality items and sold it to customers under the consideration that all the imperfect items can be remanufactured as good quality items by rework. These assumptions will underestimate the actual required quality and quantity. Hence, the defective items cannot be ignored in the production process. The primary operation strategies and goals of most manufacturing firms are to seek a high satisfaction to customer's demands and to become a low-cost producer. To reach these goals, the company should be able to effectively utilize resources and minimize costs. Rework is common in semiconductor, pharmaceutical, chemical and food industries. The products are considered as deteriorating items because their utility is lost with time of storage due to price reduction, product useful life expiration, decay and spoilage. In our lot sizing model for deteriorated items with rework, both perfect and imperfect items are deteriorating with time. The remainder of this paper is organized as follows. In section II, we give a literature review. In section III, assumptions and notations are given. The mathematical formulation for this model is given in section IV. Numerical example and sensitivity analysis are given in section V, and conclusion is drawn in section VI.

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II. LITERATURE REVIEW

Economic Production Quantity (EPQ) model is one of the prominent research topics in production, inventory control and management. By using EPQ model, optimal quantity of items and optimal production time can be obtained. Classical EPQ model was developed under various assumptions. Since then, researchers have extended the model by relaxing one or more of its assumptions. It was assumed that the items produced are of perfect quality items in the classical model. However, imperfect quality items may be produced in reality. Mukhopadhyay et al. [3] investigated an economic production quantity model for three type imperfect items with rework. Shukla et al. [4] presented an economic production quantity model with defective products for deteriorating products. Yassine et al. [5] considered disaggregating the shipments of imperfect quality items in a single production run and aggregating the shipments of imperfect items over multiple production runs. Kumar et al. [6] presented Economic Production lot size (EPLS) model with stochastic demand and shortage partial backlogging rate under imperfect quality items, in which stochastic imperfect production was assumed. Singh et al. [7] presented a mathematical production Inventory model for deteriorating items with time dependent demand rate under the effect of inflation and shortages. Rezaei et al. [8] discussed an economic production quantity and purchasing price for items with imperfect quality when inspection shifts from buyer to supplier. Felix et al. [9] presented a modified EPQ model with deteriorating production system and deteriorating Product where rework process was considered at the end of production setup. Mishra et al. [10] considered an inventory model for deteriorating items with time-dependent demand and time varying holding cost under partial backlogging. Pal et al. [11] proposed a production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness, in which, multi-production setup was considered without rework. Chandra et al. [12] introduced the effect of deterioration on two-warehouse inventory model with imperfect quality items. Kumar et al. [13] have developed a general inventory model for deteriorating items with probabilistic deterioration rate and ramp type demand under stock dependent consumption rate. Jaggi et al. [14] studied an inventory model for a retailer dealing by deteriorating items under inflationary conditions over a fixed planning horizon. Jaggi et al. [15] investigated an inventory model for non-instantaneous deteriorating items under inflationary conditions with partially backlogged shortages. A two-warehouse inventory model for deteriorating items with price dependent demand under partial backlogging was discussed by Rastogi et al. [16]. Rework process is also one important issue in reverse logistics where used products are reworked to reduce total inventory cost, waste and environmental pollution. The earliest research that focused on rework and remanufacturing process was done by Schrady [17]. Since then, researchers on rework have attracted many researchers. Khouja [18] considered direct rework for economic lot sizing and delivery scheduling problem (ELDSP). Yoo et al [19] developed an EPQ model with imperfect production, imperfect inspection and rework. Widyadana et al. [20] proposed an EPQ model for deteriorating items with rework which was performed after m production setups. Tai [21] proposed an EPQ model for deteriorating/imperfect product with rework which was performed after a production setup. Sarkara et al. [22] assumed rework for single stage production system. Hsu et al. [23] considered an EPQ model under an imperfect production process with shortages backordered. Singh et al. [24] proposed an economic production model for time dependent demand with rework and multiple production setups where production is demand dependent. Khanna [25] formulated a strategic production model to study the combined effects of imperfect quality items, inspection error and remanufacturing process under two level trade credits. We notice that not many studies considered a model with multi-production setups allowing shortages, defective items, penalty cost, scrap cost and rework. In this paper, we intend at providing analytic results to solve the issues said above.

III. ASSUMPTIONS AND NOTATIONS

A. Assumptions

1. Production and demand rate are constants.
2. Rework and deterioration rate are constants.
3. Deterioration starts as soon as it comes in the inventory.
4. There is a replacement for deteriorated items.
5. Shortages are allowed.
6. Demand during shortages is completely backordered.
7. Model is considered under finite time horizon.
8. The production rate of perfect quality items and rework must be greater than the demand rate.
9. No machine breakdown occurs in the production run and rework period.
10. Inspection cost is negligible when compare with other costs.
11. Setup time for rework process is zero.
12. All the imperfect quality items can be reproduced to good quality. No imperfect quality items occur during the rework process.

B. Notations

$D(t)$ Demand rate (unit/year)

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$P(t)$ Production rate (unit/year)
P_r Rework process rate (unit/year)
$\theta(t)$ Deterioration rate (unit/year)
α Percentage of good quality items
m Number of production setup in one cycle
p_c Penalty cost of selling deteriorated items to customers ($/unit)
c_s Shortage cost ($/unit/unit time)
K_s Production setup cost ($/setup)
K_b Production setup cost in a shortage ($/setup)
K_r Rework setup cost ($/setup)
h_s Perfect quality items holding cost ($/unit/year)
h_r Imperfect quality items holding cost ($/unit/year)
D_c Deteriorating cost ($/unit)
I_1 Inventory level of perfect quality items in a production period
I_2 Inventory level of perfect quality items in a non – production period
I_{r1} Inventory level of imperfect quality items in a production period
I_{r2} Inventory level of imperfect quality items in a non - production period
I_{r3} Inventory level of imperfect quality items in a rework production period
I_{t1} Total Inventory level of perfect quality items in a production period
I_{t2} Total Inventory level of perfect quality items in a non - production period
I_{t3} Total Inventory level of perfect quality items in a rework production period
I_{t4} Total Inventory level of perfect quality items in a rework non-production period
TTI_1 Total Inventory level of imperfect quality items in a production period
I_{v1} Total Inventory level of imperfect quality items in m production periods
TTI_2 Total Inventory level of imperfect quality items in a non-production period
I_{v2} Total Inventory level of imperfect quality items in non - production period
I_{v3} Total Inventory level of imperfect quality items in a rework setup production period
TRI items Total Inventory level of imperfect quality
IM_{r1} Maximum inventory level of recoverable items in a production setup
IM_{r2} Maximum inventory level of recoverable items in a non-production setup
IM_{r3} Maximum inventory level of recoverable items in a shortage period
IM_{r4} Maximum inventory level of recoverable items in a backlogging period
I_{Er} Maximum inventory level of imperfect quality items when rework process started
T_1 Regular production period
T_2 Non - production period
T_3 Shortage period
T_4 Complete backlogging period
T_5 Rework process period
T_6 Rework non-process period
TC Total cost per unit time

IV. FORMULATION OF THE MODEL

The behavior of the inventory level of serviceable items with three replenishment cycle and shortages is studied which is illustrated in Fig. 1. The production is performed during T_1 time period. When production is established, there are $(1-\alpha)p$ products defect per unit time. T_2 is the non-production time period. Shortages are accumulated during T_3 time period. Shortages are completely backlogged during T_4 time period.

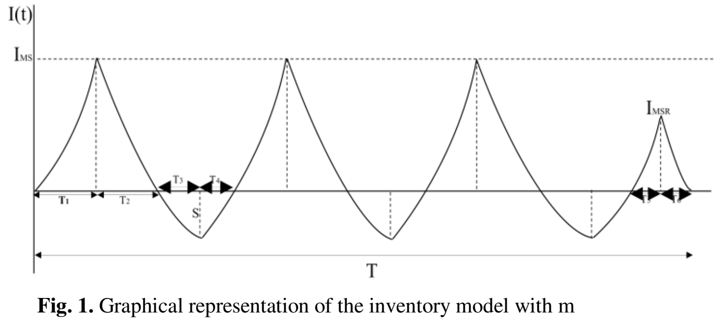

Fig. 1. Graphical representation of the inventory model with m = 3.

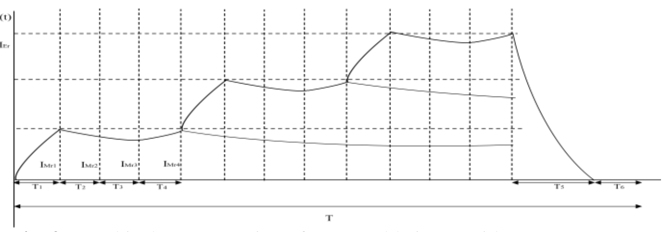

Fig. 2. Graphical representation of recoverable items with m = 3.

The inventory level of serviceable items in a production period can be formulated as:

$\frac{dI_1(t_1)}{dt_1} + \gamma\theta I_1(t_1) = \alpha p - d \qquad 0 \le t_1 \le T_1$

(1)

Since $I_1(0) = 0$, the inventory level of serviceable items in a production period is:

$I_1(t_1) = \frac{\alpha p - d}{\gamma\theta}\left[1 - e^{-\gamma\theta t_1}\right] \qquad 0 \le t_1 \le T_1$

(2)

The total inventory level of serviceable items in a production up time can be modeled as:

$I_{t1}(t_1) = \frac{\alpha p - d}{\gamma\theta}\int_0^{T_1}\left[1 - e^{-\gamma\theta t_1}\right]dt_1$

For small value of $\gamma\theta T_1$ and using Taylor series (Widyadana & Wee, [20]) one has:

$I_{t1} = \frac{(\alpha p - d){T_1}^2}{2}.$

(3)

The inventory level of serviceable item in a non-production period is represented as:

$\frac{dI_2(t_2)}{dt_2} + \theta I_2(t_2) = -d \qquad 0 \le t_2 \le T_2$

(4)

Since $I_1(T_2) = 0$ and using similar calculation setups, the total inventory in a non-production period can be formulated as:

$I_{t2} = \frac{d{T_2}^2}{2}.$

(5)

Using similar setups as above, the total serviceable inventory in a rework production period, the total serviceable inventory in a rework non-production period and their work non-production time are derived as follows:

$I_{t5} = \frac{(\alpha_r P_r - d)\,{T_5}^2}{2}.$

(6)

$I_{t6} = \frac{d\,{T_6}^2}{2}.$

(7)

The inventory level of serviceable items during a shortage non-production period is represented as:

$\frac{dI_3(t_3)}{dt_3} = -d \qquad 0 \le t_3 \le T_3$

(8)

Since $I_3(0) = 0$, the inventory level of serviceable items during a shortage non-production period is represented as:

$I_3(t_3) = -dt_3$

(9)

When $t_3 = T_3$, we obtain maximum inventory level of serviceable items during a shortage non-production period.

That is,

$I_3(T_3) = S = -dT_3$ (or) $-S = dT_3$

(10)

Total inventory level of serviceable items during a shortage non-production period is:

$I_{t3} = \frac{d\,{T_3}^2}{2}$

(11)

The inventory level of serviceable items during shortage production period is represented as:

$\frac{dI_4(t_4)}{dt_4} + \theta I_4(t_4) = -(\alpha p - d) \quad 0 \le t_4 \le T_4$

(12)

Since $I_4(0_3) = S$, we obtain the inventory level of serviceable items during shortage production period as:

$I_4(t_4) = (d - \alpha p)t_4 + S \qquad 0 \le t_4 \le T_4$

(13)

The total inventory level of serviceable items during shortage production period is:

$I_{t4} = \frac{(d - \alpha p){T_4}^2}{2}.$

(14)

The inventory level of recoverable items in a production period can be modeled as:

$\frac{dI_{r1}(t_{r1})}{dt_{r1}} + \theta I_{r1}(t_{r1}) = (1-\alpha)P \quad 0 \le t_{r1} \le T_1$

(15)

Since $I_{r1}(0) = 0$, the inventory level of recoverable items in a production period is:

$I_{r1}(t_{r1}) = \frac{(1-\alpha)p}{\gamma\theta}\left\{1 - e^{-\gamma\theta t_{r1}}\right\} \qquad 0 \le t_{r1} \le T_1$

(16)

Using Taylor series approximation, the total recoverable inventory in a production up time in one setup is:

$TTI_1 = \frac{(1-\alpha)p}{2}T_1^2.$

(17)

Since there are *m* production setups in one cycle, the total inventory for recoverable items in *m* production up time in one cycle is:

$I_{v1} = \frac{m(1-\alpha)p{T_1}^2}{2}.$

(18)

Since $I_{r1}(T_1) = I_{mr1}$, from equation (16), we get using Taylor series

$I_{mr1} = \frac{(1-\alpha)p}{\gamma\theta}\left\{T_1 - \frac{\gamma\theta T_1^2}{2}\right\}.$

(19)

The inventory level of recoverable items in a non-production period is:

$\frac{dI_{r2}(t_{r2})}{dt_{r2}} + \theta I_{r2}(t_{r2}) = 0 \qquad 0 \le t_{r2} \le T_2$

(20)

Since $I_{r2}(0) = I_{mr1}$, we get the inventory level of recoverable items in non-production period is:

$I_{r2}(t_{r2}) = I_{mr1}e^{-\gamma\theta t_{r2}} \qquad 0 \le t_{r2} \le T_2$

(21)

Using Taylor series expansion, the total recoverable inventory in a non-production period in one setup is:

$TTI_2 = I_{mr1}\left\{T_2 - \frac{\gamma\theta T_2^2}{2}\right\}$

(22)

Since there are *m* non-production setups in one cycle,

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the total inventory for recoverable items in *m* non-production period in one cycle is:

$$I_{v2} = mI_{mr1}\left\{T_2 - \frac{\gamma\theta T_2^2}{2}\right\}$$
(23)

Since $I_{r2}(T_2) = I_{mr2}$,

$$I_{mr2} = I_{mr1}\left[1 - \gamma\theta T_2 + \frac{(\gamma\theta T_2)^2}{2}\right]$$
(24)

Using similar steps as above, the total inventory level of recoverable items in shortage and shortage backlogging period in one cycle and maximum inventory level of recoverable items in a shortage and shortage backlogging period are derived as follows:

$$I_{v3} = mI_{mr2}\left\{T_3 - \frac{\gamma\theta T_3^2}{2}\right\}.$$
(25)

$$I_{mr3} = I_{mr2}\left[1 - \gamma\theta T_3 + \frac{(\gamma\theta T_3)^2}{2}\right]$$
(26)

$$I_{v4} = mI_{mr4} + [\gamma\theta I_{mr3} + (1-\alpha)p]\frac{mT_4^2}{2}.$$
(27)

$$I_{mr4} = I_{mr3} + [(1-\alpha)p - \gamma\theta I_{mr3}]T_4 + [(\gamma\theta)^2 I_{mr3} - (1-\alpha)p\gamma\theta]\frac{T_4^2}{2}.$$
(28)

The inventory level of recoverable items in *(m – 1)* non-production period is:

$$\frac{dI_{r(m-1)}(t_{r(m-1)})}{dt_{r(m-1)}} + \theta I_{r(m-1)}(t_{r(m-1)}) = 0$$
$$0 \le t_{r(m-1)} \le (m-1)(T_1 + T_2 + T_3 + T_4)$$
(29)

Since $I_{r(m-1)}(0) = I_{mr4}$,

$$I_{r(m-1)}(t_{r(m-1)}) = I_{mr4}e^{-\gamma\theta t_{r(m-1)}}$$
$$0 \le t_{r(m-1)} \le (m-1)(T_1 + T_2 + T_3 + T_4)$$
(30)

The inventory level of recoverable items in *(m-1)* non-production periods can be modeled as follows:

$$I_{v(m-1)} = \sum_{k=1}^{m}\int_{t_{r(m-1)}=0}^{(k-1)(T_1+T_2+T_3+T_4)} I_{mr4}\, e^{-\gamma\theta t_{r(m-1)}}\, dt_{r(m-1)}$$

Using Taylor series expansion, one can get:

$$I_{v(m-1)} = \sum_{k=1}^{m} I_{mr4}\left[(k-1)(T_1 + T_2 + T_3 + T_4) - \frac{\gamma\theta[(k-1)(T_1+T_2+T_3+T_4)]^2}{2}\right].$$
(31)

Since $I_{r(m-1)}[(m-1)(T_1 + T_2 + T_3 + T_4)] = I_{Er}$, from equation (30), we get

$$I_{Er} = \sum_{k=1}^{m} I_{mr4}e^{-\gamma\theta(m-1)(T_1+T_2+T_3+T_4)}$$

Using Taylor series expansion, we get

$$I_{Er} = \sum_{k=1}^{m} I_{mr4}\left[1 - \gamma\theta(m-1)(T_1 + T_2 + T_3 + T_4) + \frac{[\gamma\theta(m-1)(T_1+T_2+T_3+T_4)]^2}{2}\right]$$
(32)

The inventory level of recoverable item in a rework period can be formulated as:

$$\frac{dI_{r5}(t_{r5})}{dt_{r5}} + \theta I_{r5}(t_{r5}) = -p_r \qquad 0 \le t_{r5} \le T_5$$
(33)

Solving equation (33), we get the inventory level of recoverable item in a rework period as:

$$I_{r5}(t_{r5}) = \frac{p_r}{\gamma\theta}\left[e^{\gamma\theta(T_5 - t_{r5})} - 1\right] \qquad 0 \le t_{r5} \le T_5$$
(34)

The total inventory of recoverable items in a rework period can be modeled as:

$$I_{v5} = \frac{p_r}{2}T_5^2.$$
(35)

In order to find the optimal cycle length and the optimal time periods T_1, T_2, T_3, T_4 , T_5 and T_6 , we use the following method for getting an approximation of TC for which the optimal solution can be obtained. Such approximation is commonly used in modeling systems for deteriorating items; see for example (Widyadana and Wee, [20]; Pal et al. [11]). We set T_4 as decision variable and $T_1 = lT_4$.

Since $I_1(T_1) = I_2(0)$, we get the non-production time period T_2 as:

$$T_2 \cong \frac{(\alpha p - d)[2T_1 - (\gamma\theta)T_1{}^2]}{2d}$$
(36)

Since $I_5(T_5) = I_6(0)$, we get the rework non-production time period T_6 as:

$$T_6 \cong \frac{(\alpha_r p_r - d)[2T_5 - (\gamma\theta)T_5{}^2]}{2d}$$
(37)

Since $I_{r5}(0) = I_{Er}$, from equation (33) we get the maximum inventory level of recoverable items when rework process started as:

$$I_{Er} = \frac{p_r}{\gamma\theta}\left[e^{\gamma\theta T_5} - 1\right]$$

Since θT_5 is very small and using Taylor series expansion we get the rework process time period T_5 as:

$$T_5 = \frac{I_{Er}}{p_r}.$$
(38)

Since $S = dT_3 = (\alpha p - d)T_4$ we obtain the Shortage accumulated time period T_3 and Shortage backlogging time period T_4 as:

$$T_3 = \frac{S}{d}.$$
(39)

Now the relation between the time periods is given by $T = T_1 + T_2 + T_3 + T_4 + T_5 + T_6$.

Total objective function of the proposed model is determined as follows:

TRI = Total recoverable inventory in a cycle
$= I_{v1} + I_{v2} + I_{v3} + I_{v4} + I_{v5}$.

TDI = Total deteriorating units in a cycle
$= (m\alpha pT_1 - mdT_1) + (m\alpha pT_4 - mdT_4) + (\alpha_r p_r T_5 - dT_5) - mdT_2 - dT_6$

PC = Penalty cost of selling deteriorated items to customers

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$$= \frac{1-\gamma}{\gamma}\{mp_c[(\alpha p - d)T_1 - I_{ms}] + mp_c[I_{ms} - dT_2] + p_c[(\alpha_r p_r - d)T_5 - I_{msr}] + p_c[I_{msr} - dT_6]\}$$

TC = Production set up cost + Rework setup cost + Holding cost of serviceable items + Holding cost of recoverable items + Deteriorating cost + Penalty cost + Scrap cost + Shortage cost.

$$TC(m,T_4) = \frac{1}{T}\begin{bmatrix} m(k_s + k_b) + k_r + h_s(mI_{t1} + mI_{t2} + I_{t5} + I_{t6}) + h_r(TRI) \\ +D_c(TDI) + PC + S_c(1-\alpha_r)p_r T_5 + C_s(I_{t3} + I_{t4}) \end{bmatrix} \quad (40)$$

where $T = T_1 + T_2 + T_3 + T_4 + T_5 + T_6$.

To minimize the total cost $TC(m,T_4)$ per unit time, the optimal value of m and T_4 can be obtained by solving the following equations:

$$\frac{\partial[TC(m,T_4)]}{\partial T_4} = 0 \text{ and } \frac{\partial[TC(m,T_4)]}{\partial m} = 0 \quad (41)$$

providing that equation (40) satisfies the following conditions:

$$\left(\frac{\partial^2[TC(m,T_4)]}{\partial {T_4}^2}\right)\left(\frac{\partial^2[TC(m,T_4)]}{\partial m^2}\right) - \left(\frac{\partial^2[TC(m,T_4)]}{\partial T_4 \partial m}\right)^2 > 0$$

and $\frac{\partial^2[TC(m,T_4)]}{\partial {T_4}^2} > 0$

By solving (41), the value of T_4 and m can be obtained and using this values in equation (40), the minimum total inventory cost per unit time of the inventory system is derived. Since the nature of the cost function is nonlinear, the convexity of the function is derived using maple mathematical software in the next section.

V. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

In this section, a numerical example and sensitivity analysis are given to illustrate the proposed model.

A. Numerical example

The above model is demonstrated by the following numerical example. Consider the following parameter values: $p = 5000$, $d = 700$, $p_r = 1000$, $\alpha = 0.8$, $\alpha_r = 0.8$, $\gamma = 0.6$,
$\theta = 0.15$, $k_s = 20$, $k_b = 10$, $k_r = 50$, $h_s = 200$, $h_r = 30$, $D_c = 15$, $s_c = 200$, $c_s = 120$, $p_c = 50$.
We obtain the following optimal time length and quantity:
$T_1^* = 0.101$, $T_4^* = 0.380$ and $Q^* = (T_1 + T_4)p = 2405$
and the optimum total cost with $m^* = 7$ is $TC^* = \$\ 136300$.

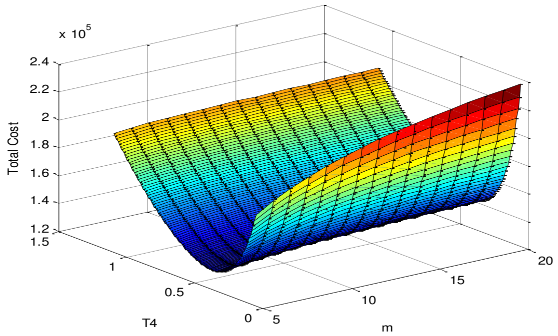

Fig. 3: Convexity of total cost function with m and T_4

B. Sensitivity analysis

The sensitivity analysis is performed by changing the values of a parameter of this model and keeping other parameters unchanged. We can obtain the following observations from Fig. 4 to Fig. 8:

1. No. of replenishment (m) increases the total cost function (TC) is also increasing. The total cost function is very highly sensitive with the first two replenishment cycle after that the function is lightly sensitive. To reduce the total cost, the manufacturer must perform minimum number of replenishment.
2. As we increase the value of deteriorating rate (θ), the total cost first decreases and then increases gradually. So the total cost is moderately sensitive with the total cost function (TC).
3. When we increase the holding cost of serviceable items (h_s), the total cost function is also increasing. The total cost function is highly sensitive with the holding cost of serviceable items (h_s). So, to reduce the total inventory cost, the manufacturer should store the very less deteriorating rate items.
4. As we increase the value of scrap cost (c_s), the total cost function (TC) is also increasing. The total cost function is highly sensitive with the scrap cost. So, to reduce the total inventory cost, the manufacturer should reduce the production of scrap items.
5. As we increase the value of α, the total cost function is gradually decreasing. The total cost function is highly sensitive with α. It is obvious that reducing of production of defective items leads to reduce the total inventory cost. To reduce the total inventory cost, the manufacturer should reduce the production of defective items.

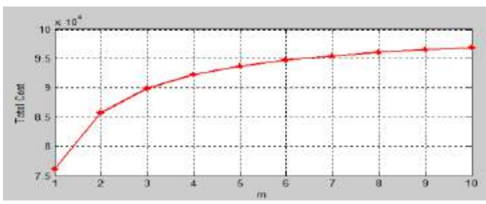

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Fig. 4: m v/s Total cost

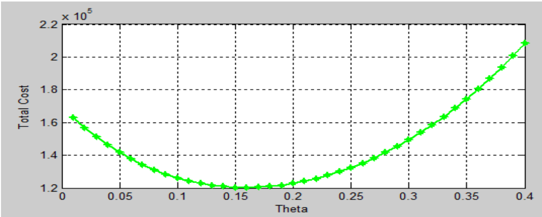

Fig. 5: θ v/s Total cost

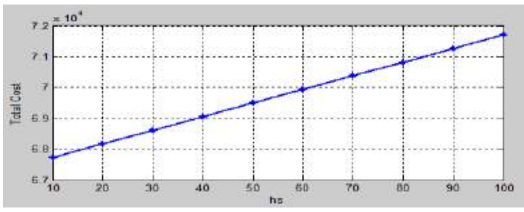

Fig. 6: h_s v/s Total cost

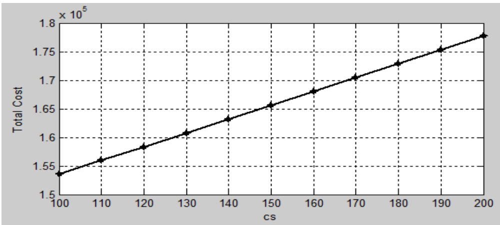

Fig. 7: c_s v/s Total cost

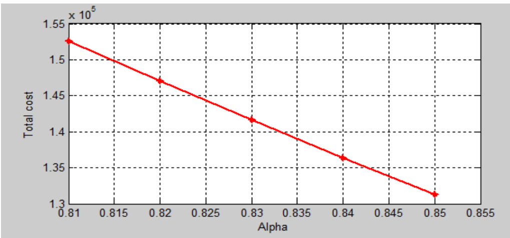

Fig. 8: α v/s Total cost

VI. Conclusion

In this article we introduced an inventory model for defective and deteriorating items in which rework and shortages are considered under multi-replenishment cycle. We have developed this model remembering that these days managing defective things has turned into an essential and developing range of research and rework takes major role to green production system. The model can be used in textile industries, footwear, chemical, food, cosmetics, etc. where the defective items will be produced in each cycle of production. Internally deteriorating items are another significant issues since which are not able to screen out. The shortages are allowed and accumulated ~hortages are completely backlogged. This model ɜlps manufacturer to obtain optimum ꞉plenishment run time, optimum production ɹantity and optimum cycle length. The sensitivity ɿows that the purchase cost, holding cost and ꞉rap cost much affect the proposed model. The ꞉oposed model can be extended in several ways. or instance, we may extend the constant ..eterioration rate to a time dependent deterioration rate. We could consider the demand as a function of ling price, stock-dependent, etc. Finally, we ɿld generalize the model for quantity discount ɿ others.

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