Integrated Optimal Solution for Probabilistic Deteriorating system and Exponential Increasing Holding Cost of Vendor - Buyer

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Abstract - An incorporated most suitable policy for supplier and customer is studied while units in inventory are difficulty to deterioration at one-of-a-kind rates and protecting cost is exponentially increasing feature. here we assumed deteriorating gadgets regarding probabilistic deterioration in which the demand is depending on sales group's tasks. A best policy that minimizes the overall fee is developed. The goal of this examine as to remember one-of-a-kind types of continuous probabilistic deterioration features and to locate the related general fee. to demonstrate the proposed version numerical example is given. Sensitivity analysis of the superior answers with appreciate to important parameters are achieved.

Keywords Integrated Optimal Strategy, Permissible delay period, Probability deterioration

1 Introduction

In formulating stock fashions deterioration of items need to receive due consideration. objects like food, pharmaceutical, chemicals, and so on. go to pot significantly. To manipulate the inventory degree efficaciously, the store needs to find a balance among the fees and advantages of maintaining inventory. The charges of holding stock consist of the money has been spent shopping for the stock in addition to storage. The blessings include having sufficient inventory handy to meet the demand of customers. Having an excessive amount of inventory equals more price for the store as it could lead to a shortfall in cash glide and incur excess storage prices. And having too little inventory equals lost profits in the form of lost income, whilst additionally undermining customer self-belief in store's capability to deliver the products the store claims to promote. consequently, maintaining the proper stock and being able to promote it may lead to increased income, new clients, extended customer self-assurance, progressed coins waft.

In this article, an integrated vendor- buyer inventory system is studied when units in inventory deteriorate at different rate. A negotiation factor is incorporates to share the cost savings. [1]Ajai et al. developed a mathematical model for an optimal ordering policy in the joint strategy of vendor-buyer inventory system when demand is quadratic. [3]developed a time-dependent quadratic inventory model for deteriorating items with permissible delay in payments. Also shortages are allowed and partially backlogged.[4]Palanivel and Uthayakumar developed an economic production lot size inventory model for deteriorating items with the consideration of sale's team initia- tive dependent demand and variable production rate. [11]Sharmila and Uthayakumar developed a mathematical model of an inventory system in which demand depending upon stock level and time with various degree β, gave more flexibility of the demand pattern and more general to the study done so far with the condition to minimize the total average cost of the system. [12] Sharmila and Uthayakumar presented a mathematical model of an inventory system in which demand depending upon stock level and time with various value of β.[10]Sharmila and Uthayakumar examined the partial trade credit financing in a supply chain by EOQ-based model for decomposing items together with shortages.[9]Sharmila and Uthayakumar presented fuzzy inventory model for deteriorating items with short- ages under fully backlogged condition.[13]Sundara Rajan and Uthayakumar considered two different cases to obtain optimal total profit, optimal order quantity and optimal replenishment policy. [8]Sinngh and Singh devoted to products having power demand pattern with variable rate of deterioration under the effect of inflation.

[14]Umakanta presented a production inventory model for controllable probabilistic deterioration rates in which three different levels of production.[7]Ravish assumed an inventory model with exponential declining demand and variable deterioration is to be optimized. Also holding cost is a linear function of time. [2]Bhanu et al. developed which investigates the optimal order quantity of the on-hand inventory due to an exponential declining demand rate. [5]Preeti and Singh addressed a finite time-horizon production-inventory model for deteriorating items having exponential demand rate under shortage cost and deterioration cost in fuzzy environment where both the perfect and imperfect items are produced. [6]presented a multi - item reverse logistics inventory model for integrated production of new items and re manufacturing of defective and returned items.

2 Notations and Assumptions

The following notations and assumptions are used in this paper.

2.1 Notations

A_b	Buyer's ordering cost per order
A_v	Vendor's ordering cost per order
C_b	Buyer's purchase cost per order
C_v	Vendor's purchase cost per order
I_b	Inventory carrying charge fraction per unit per time unit for vendor I_v
	Inventory carrying charge fraction per unit per time unit for vendor θ_b

	Deterioration of items in vendor's inventory system, $0 < \theta_v < \theta_b < 1$
$I_b(t)$	Buyer's inventory level at instant of time t
$I_v(t)$	vendor-buyer combined inventory level at instant of time t
n	number of time of orders kept by buyer during cycle time
TC_b	Buyer's total cost per time unit
TC_v	Vendor's total cost per time unit
TC	Integrated total cost for both vendor and buyer per unit time
T	Vendor's cycle time
M	Permissible delay period offered by the vendor to the buyer
r	continuous rate

2.2 Assumptions

1. $R(t) = ab^m$ demand rate at the time t, where $a > 0$, $b > 0$ $m = 1, 2, 3...$
2. The holding cost is time dependent and $h(t) = fexp(dt)$ where f and g are positive constant.
3. An inventory system of single vendor and single buyer is considered.
4. Lead time is zero.
5. The deterioration follows continuous probability distribution function as (a) Uniform (b) Triangular (c) Beta.
6. There is no repair or replacement of deteriorated units during a cycle time.
7. Time horizon is infinite.
8. Shortages are not allowed.

3 Mathematical Formulation

Figure 1 represents time-varying inventory status for the vendor and the buyer. The inventory depletes due to demand and deterioration rate for both vendor and buyer. The rate of change of inventory for the vendor and buyer is given by the differential equation

$$\frac{dI_b(t)}{dt} + \theta_b I_b(t) = -ab^m, 0 \leq t \leq \frac{T}{n} \tag{1}$$

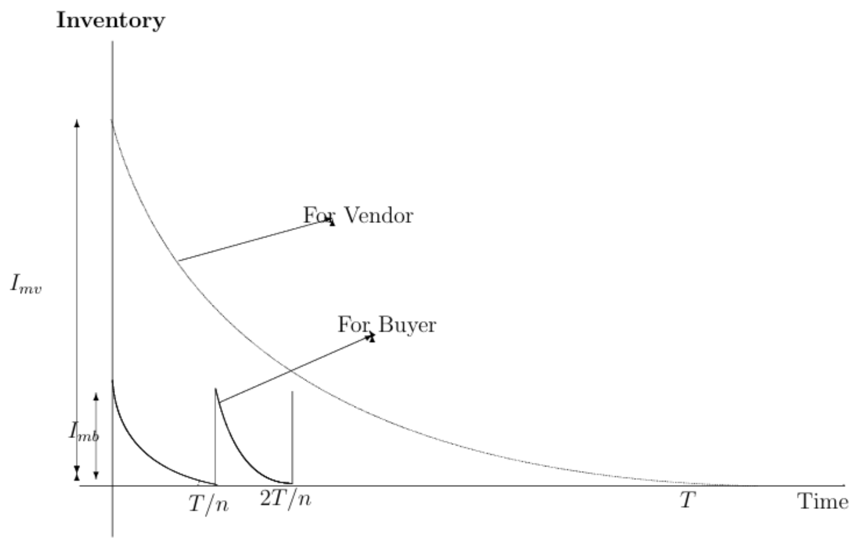


Figure 1: Time-Inventory Status of Vendor and Buyer

and

$$\frac{dI_v(t)}{dt} + \theta_v I_v(t) = -ab^m, 0 \le t \le T \tag{2}$$

with boundary conditions $I_b(\frac{T}{n}) = 0$ and $I_v(T) = 0$. $I_b(0) = I_{mb}$ and $I_v(0) = I_{mv}$. The solution of the differential equation are,

$$I_b(t) = \frac{ab^m}{\theta_b}\left[e^{\theta_b\left(\frac{T}{n}-t\right)} - 1\right] \tag{3}$$

$$I_v(t) = \frac{ab^m}{\theta_v}\left[e^{\theta_v(T-t)} - 1\right]$$

Using $I_b(0) = I_{mb}$ and $I_v(0) = I_{mv}$, the purchase quantities for the buyers and the vendor are

$$I_{mb} = \frac{ab^m}{\theta_b}\left[e^{\frac{\theta_b T}{n}} - 1\right]$$
$$I_{mv} = \frac{ab^m}{\theta_v}\left[e^{\theta_v T} - 1\right]$$

Using $I_b(0) = I_{mb}$ and $I_v(0) = I_{mv}$

$$I_{mb} = \frac{ab^m}{\theta_b}\left[e^{\frac{\theta_b T}{n}} - 1\right]$$
$$I_{mv} = \frac{ab^m}{\theta_v}\left[e^{\theta_v T} - 1\right]$$

During the cycle time $[0, T]$, the buyer's holding cost is

$$IHC_b = \int_0^{\frac{T}{n}} h(t) I_b(t) dt$$
$$IHC_b = fab^m\left[\frac{e^{\theta_b\frac{T}{n}+t(d-\theta_b)}}{d-\theta_b} - \frac{e^{dt}}{d}\right]$$

Ordering cost is $OC_b = nA_b$ and number units deteriorated is $\left(I_{mb} - \frac{T}{n}R\left(\frac{T}{n}\right)\right)$. Hence, the buyer's due to deterioration of units is

$$CD_b = nC_b\left(I_{mb} - \frac{T}{n}R\left(\frac{T}{n}\right)\right)$$

Hence the buyer's total cost TC_b per time unit is

$$TC_b = \frac{1}{T}[IHC_b + CD_b + OC_b]$$

$$TC_b = \frac{1}{T}\left[fab^m\left[\frac{e^{\theta_b\frac{T}{n}+t(d-\theta_b)}}{d-\theta_b} - \frac{e^{dt}}{d}\right] + nC_b\left(I_{mb} - \frac{T}{n}R\left(\frac{T}{n}\right)\right) + nA_b\right]$$

The vendor's inventory is the joint two echelon inventory model is the difference between the vendor-buyer combined inventory and the buyer's inventory. Therefore, vendor's holding cost is

$$IHC_v = f\exp(dt)\left[\int_0^T I_v(t)dt - n\int_0^{\frac{T}{n}} I_b(t)dt\right]$$

$$IHC_v = \frac{fab^m}{\theta_v}\left[\frac{e^{T(d-\theta_v)+\theta_v T}}{d-\theta_v} - \frac{e^{dt}}{d} - \frac{e^{\theta_v T}}{d-\theta_v} + \frac{1}{d}\right] + \frac{nfab^m}{\theta_b}\left[\frac{e^{\frac{T(d-\theta_b)}{n}+\frac{T\theta_b}{n}}}{d-\theta_b} - \frac{e^{dT}}{d} - \frac{e^{\frac{T\theta_b}{n}}}{d-\theta_b} + \frac{1}{d}\right]$$

The units deteriorated at vendor's inventory system is $(I_{mv} - nI_{mb})$. and vendor's ordering cost is $OC_v = A_v$

The vendor's total cost TC_v per time unit is

$$TC_v = \frac{1}{T}[IHC_v + CD_v + OC_v]$$

$$TC_b = \frac{1}{T}\left[\frac{fab^m}{\theta_v}\left[\frac{e^{T(d-\theta_v)+\theta_v T}}{d-\theta_v} - \frac{e^{dt}}{d} - \frac{e^{\theta_v T}}{d-\theta_v} + \frac{1}{d}\right] + \frac{nfab^m}{\theta_b}\left[\frac{e^{\frac{T(d-\theta_b)}{n}+\frac{T\theta_b}{n}}}{d-\theta_b} - \frac{e^{dT}}{d} - \frac{e^{\frac{T\theta_b}{n}}}{d-\theta_b} + \frac{1}{d}\right] + A_v + (I_{mv} - nI_{mb})\right]$$

The joint total cost TC is the sum of TC_b and TC_v. Since $T_b = \frac{T}{n}$, TC is the function of discrete variable n and continuous variable T.

3.1 Deterioration follows uniform distribution

We consider that the deterioration θ follows uniform distribution as $\theta = E[f(x)] = \frac{\alpha+\beta}{2}, \alpha > 0, \beta > 0, \alpha < \beta$. Therefore, the equation of

$$TC_b = \frac{1}{T}\left[fab^m\left(\frac{e^{\frac{K_bT}{n}+t(d-K_b)}}{d-K_b} - \frac{e^{dt}}{d}\right) + nC_b\left(I_{mb} - \frac{T}{n}R\left(\frac{T}{n}\right)\right) + nA_b\right]$$

$$TC_v = \frac{1}{T}\left[\frac{nfab^m}{K_v}\left(\frac{e^{T(d-K_v)+K_vT}}{d-K_v} - \frac{e^{dt}}{d} - \frac{e^{K_vT}}{d-K_v} + \frac{1}{d}\right)\right] + \frac{nfab^m}{K_b}\left[\frac{e^{\frac{T(d-K_b)}{n}+\frac{TK_b}{n}}}{d-K_b} - \frac{e^{\frac{dT}{n}}}{d} - \frac{e^{\frac{TK_b}{n}}}{d-K_b} + \frac{1}{d}\right] + C_V\left[\frac{ab^m}{K_v}(e^{K_vT}-1) - n\left(\frac{ab^m}{K_b}\left(e^{\frac{K_bT}{n}}-1\right)\right) + A_v\right]$$

Where $K_b = \frac{\alpha+\beta}{2}, K_v = \frac{\alpha+\beta}{2}$

3.2 Deterioration follows triangular distribution

We consider that deterioration θ follows triangular distribution as $\theta = E[f(x)] = \frac{\alpha+\beta+\gamma}{3}$ where $f(x)$ is the probability density function of the triangular distribution with lower limit α, upper limit β and mode γ as well as $\alpha < \beta$ and $\alpha \le \gamma \le \beta$. Therefore, the equation of

$$TC_b = \frac{1}{T}\left[fab^m\left(\frac{e^{\frac{K_bT}{n}+t(d-K_b)}}{d-K_b} - \frac{e^{dt}}{d}\right) + nC_b\left(I_{mb} - \frac{T}{n}R\left(\frac{T}{n}\right)\right) + nA_b\right]$$

$$TC_v = \frac{1}{T}\left[\frac{nfab^m}{K_v}\left(\frac{e^{T(d-K_v)+K_vT}}{d-K_v} - \frac{e^{dt}}{d} - \frac{e^{K_vT}}{d-K_v} + \frac{1}{d}\right)\right] + \frac{nfab^m}{K_b}\left[\frac{e^{\frac{T(d-K_b)}{n}+\frac{TK_b}{n}}}{d-K_b} - \frac{e^{\frac{dT}{n}}}{d} - \frac{e^{\frac{TK_b}{n}}}{d-K_b} + \frac{1}{d}\right] + C_V\left[\frac{ab^m}{K_v}(e^{K_vT}-1) - n\left(\frac{ab^m}{K_b}\left(e^{\frac{K_bT}{n}}-1\right)\right) + A_v\right]$$

Where $K_b = \frac{\alpha+\beta+\gamma}{3}, K_v = \frac{\alpha+\beta+\gamma}{3}$

3.3 Deterioration follows beta distribution

We consider that deterioration θ follows beta distribution as $\theta = E[f(x)] = \frac{\alpha}{\alpha+\beta}$ where $f(x)$ follows a continuous probability distribution defined on the interval $(0,1)$ parameterized by two positive parameters denoted by α and β. Therefore, the equation of

TC can be written as

$$TC_b = \frac{1}{T}\left[fab^m\left(\frac{e^{\frac{K_bT}{n}+t(d-K_b)}}{d-K_b} - \frac{e^{dt}}{d}\right) + nC_b\left(I_{mb} - \frac{T}{n}R\left(\frac{T}{n}\right)\right) + nA_b\right]$$

$$TC_v = \frac{1}{T}\left[\frac{nfab^m}{K_v}\left(\frac{e^{T(d-K_v)+K_vT}}{d-K_v} - \frac{e^{dt}}{d} - \frac{e^{K_vT}}{d-K_v} + \frac{1}{d}\right)\right]$$

$$+\frac{nfab^m}{K_b}\left[\frac{e^{\frac{T(d-K_b)}{n}+\frac{TK_b}{n}}}{d-K_b} - \frac{e^{\frac{dT}{n}}}{d} - \frac{e^{\frac{TK_b}{n}}}{d-K_b} + \frac{1}{d}\right] +$$

$$C_V\left[\frac{ab^m}{K_v}(e^{K_vT}-1) - n\left(\frac{ab^m}{K_b}\left(e^{\frac{K_bT}{n}}-1\right)\right) + A_v\right]$$

Where $K_b = \frac{\alpha}{\alpha+\beta}, K_v = \frac{\alpha}{\alpha+\beta}$

4 Computation Procedure

There are two cases:
Case 1:
When vendor and buyer make decision independently.
For given value of n, TC_b can be minimized by solving $\frac{\partial TC_b}{\partial T_b} = 0$ for T_b.
This solution (n, T_b) minimizes TC_v provided

$$TC_v(n-1) \geq TC_v(n) \leq TC_v(n+1)$$

gets satisfied. Then the total cost without integration, TC_{NJ} is given by,

$$TC_{NJ} = \min_n\left[\min_n TC_b + TC_v\right]$$

Case 2:
When vendor and buyer make decision jointly.
The optimum values of T and n must satisfy the following condition simultaneously:

$$\frac{\partial TC}{\partial T} = 0 \text{ and } TC(n-1) \geq TC(n) \leq TC(n+1)$$

The total integrated cost is

$$TC_J = \min_{T,n}[TC_b + TC_v]$$

Clearly $TC_J \leq TC_{NJ}$.Hence, total cost savings, Sav_J is defined as $Sav_J = TC_{NJ} - TC_J$. Let the buyer's cost saving, Sav_b be defined as $Sav_b = vSav_J$, where ϑ is the negotiation factor and $0 \leq \vartheta \leq 1$. When negotiation factor equal to 1 , all saving goes to buyer; When negotiation factor is 0.5 , total savings is equally distributes between the vendor and the buyer. The present value of unit cost after a time interval M permissible credit period is e^{-rg} where r is discounting rate. Solving the following equation

$$R(T)C_b(1 - e^{-rg}) = Sav_b$$

The buyer's permissible delay in payment can be computed as

$$M = \frac{1}{r}\ln\left[\frac{C_bR(T)}{C_bR(T) - Sav_b}\right]$$

In our model, we consider the deterioration θ which follows three different types of probability distribution function as $\theta = E[f(x)]$ where $f(x)$ follows (i) uniform distribution (ii) Triangular distribution (iii) Beta distribution. We show a numerical comparison between the three models.

5 Numerical Results and Discussion

To illustrate proposed model, consider following parameter values
Example 1

$$a = 200, b = 0.4, m = 1, A_b = 600, A_v = 300, C_b = 25, C_v = 15, I_b = 0.11, I_v = 0.10, \theta_b = 0.20, \theta_v = 0.10, r = 0.03, f = 15, d = 8, s = 50, p = 25, \alpha = 0.10, \beta = 0.30, \gamma = 0.20, t = 0.523, T = 0.879$$

Total Cost for Buyer=RS.2692.7
Total Cost for Vendor=RS.341.2969
Example 2 The data are same as in Example 1

Table 1: Numerical Example (1)

θ	t_1	T	TC_b	TC_v
Beta	0.5373	0.6192	2981.0	484.4961
Triangular	0.5241	0.6743	2901.3	444.9058
Uniform	0.5047	0.7386	2832.6	406.1738

Table 2: Numerical Example (2) Beta

α	β	t_1	T	TC_b	TC_v
0.20	0.40	0.4686	0.6604	2939.1	454.2701
0.30	0.50	0.5394	0.6246	3011.4	480.3074
0.40	0.60	0.5178	0.5936	3082.6	505.3901
0.50	0.70	0.5388	0.6261	3060.0	479.1551
0.60	0.80	0.5170	0.5948	3123.0	504.3712
0.70	0.90	0.7592	0.8656	2820.7	346.584

Table 3: Numerical Example (2) Triangular

α	β	γ	t_1	T	TC_b	TC_v
0.10	0.20	0.30	0.4686	0.6604	2920.1	454.2701
0.20	0.30	0.40	0.5394	0.6246	2991.7	480.3074
0.30	0.40	0.50	0.5178	0.5936	3062.2	505.3908
0.40	0.50	0.60	0.5388	0.6261	3029.3	479.1567
0.50	0.60	0.70	0.5170	0.5948	3101.5	504.3712
0.60	0.70	0.80	0.7592	0.8656	2799.8	346.5804
0.70	0.80	0.90	0.8251	0.9136	2783.3	328.3713

Table 4: Numerical Example (2) Uniform

α	β	t_1	T	TC_b	TC_v
0.10	0.30	0.4686	0.6604	4080.5	454.2701
0.20	0.40	0.5394	0.6246	4158.5	480.3074
0.30	0.50	0.5178	0.5936	4222.4	505.3908
0.40	0.60	0.5388	0.6261	4176.7	479.1567
0.50	0.70	0.5170	0.5948	4233.4	504.3712
0.60	0.80	0.7592	0.8656	3914.6	346.5804
0.70	0.90	0.8251	0.9136	3880.1	328.3713
0.80	0.90	0.8623	0.9654	3853.9	310.7520

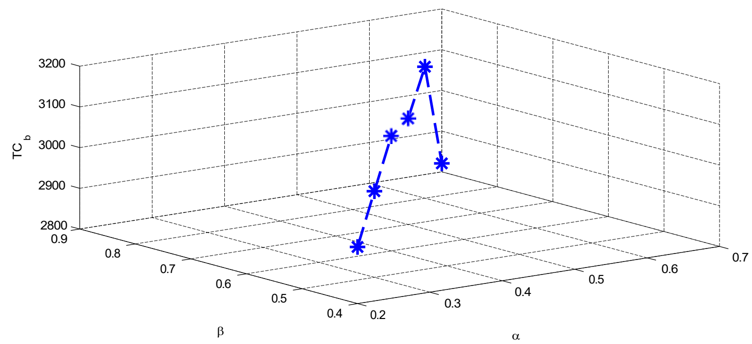


Figure 2: Variation of optimal time α and β to get the buyer's total cost

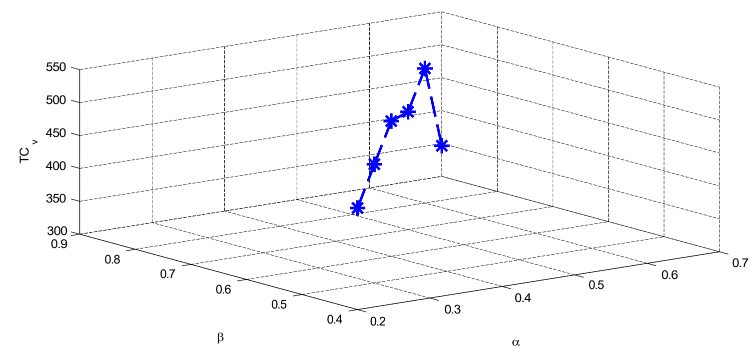
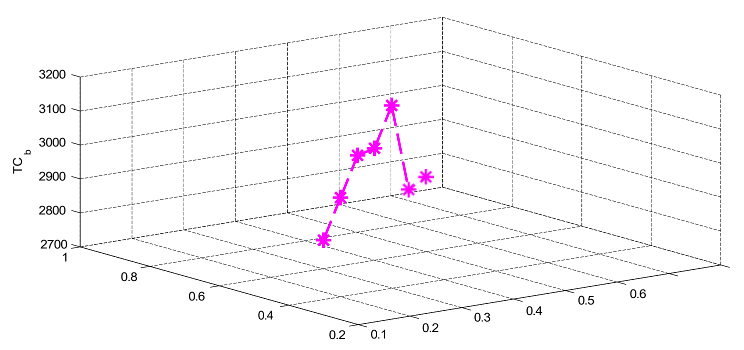


Figure 3: Variation of optimal time α and β to get the vendor's total cost

Figure 4: Variation of optimal time α and γ to get the buyer's total cost



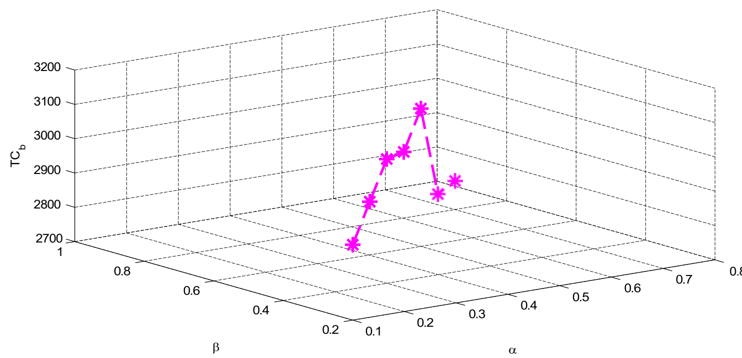


Figure 5: Variation of optimal time α and β to get the buyer's total cost

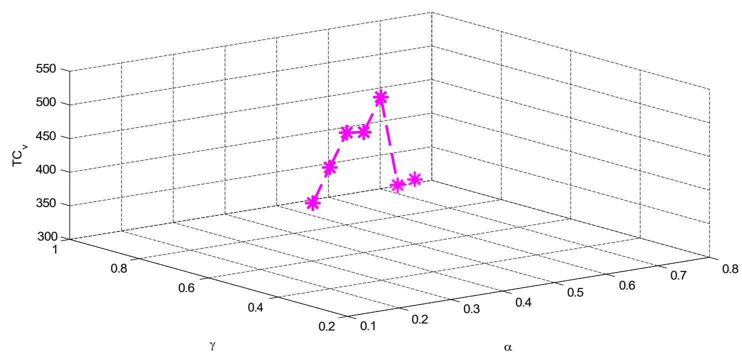


Figure 6: Variation of optimal time α and γ to get the vendor's total cost

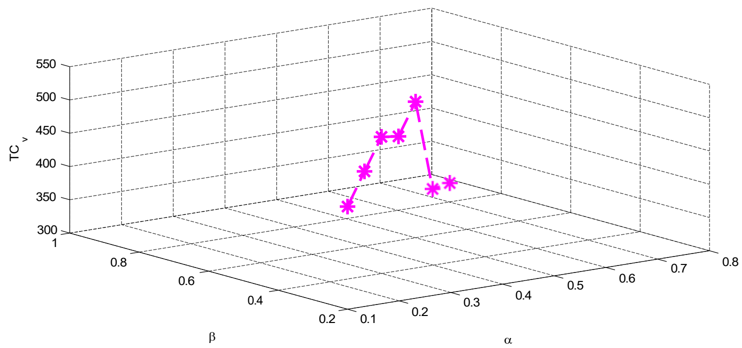


Figure 7: Variation of optimal time α and β to get the vendor's total cost

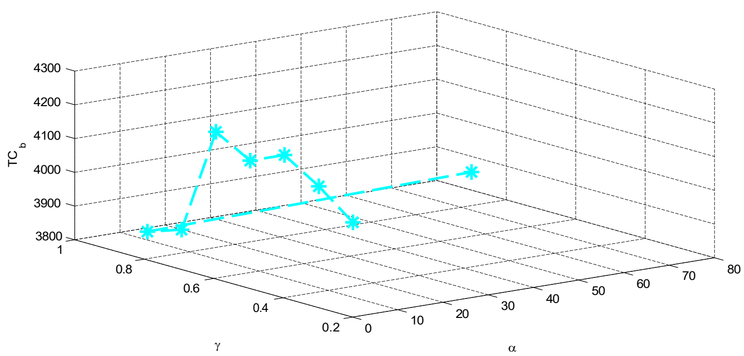


Figure 8: Variation of optimal time α and γ to get the buyer's total cost

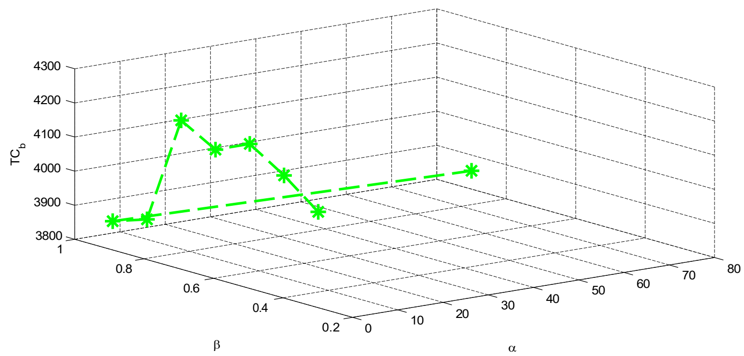


Figure 9: Variation of optimal time α and γ to get the buyer's total cost

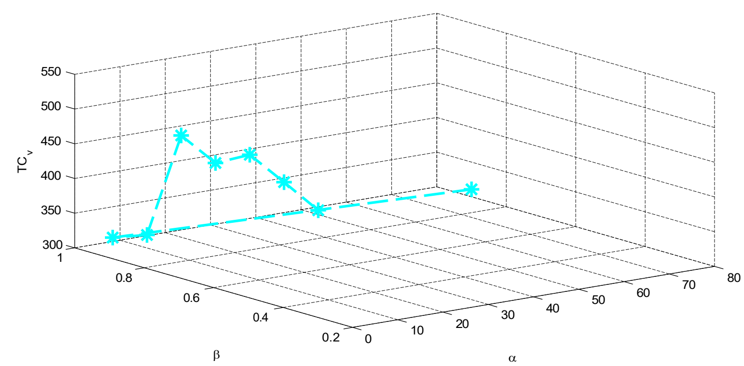


Figure 10: Variation of optimal time α and β to get the vendor's total cost

Figure 11: Variation of optimal time α and γ to get the vendor's total cost

6 Conclusion

In this study, a mathematical model is developed for an optimal ordering policy in the joint strategy of vendor-buyer inventory system. An analytic formulation of the model and an optimal solution procedure to find the optimal replenishment policy were also presented. Moreover sensitivity analysis of the optimal solutions with various parameters were carried out. Graphical representation is provided to establish the total profit function attains it's optimum value. A possible future research issue is to consider two level trade credit or trade credit linked with order quantity with time exponential holding cost. A similar comparative study could be done by developing a multi - echelon economic order quantity model by considering inflation and delay in payments.

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